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The Network Equilibrium Problem with Mixed Demand

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Abstract—We formulate the network equilibrium problem with mixed demand which generalizes the problems of network equilibrium with fixed and elastic demand. We prove the equilibrium conditions for this problem and propose some conditions of existence of a solution that are based on the coercivity property. We establish a connection between the problem of network equilibrium with mixed demand and the problem of auction equilibrium. The results of test calculations are presented for a model example.

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INTRODUCTION

The network equilibrium problems arise in various areas of human activity including transport and telecommunication networks (see, for example, [6, 7, 11—13]). Their study began already in the 1950s [2], whereas in the form of variational inequalities these problems were formulated in the early 1980s in [4, 15] for the fixed demand and in [5] for the elastic demand. It is surprising that over the following 35 years, as far as the author knows, there were no attempts to combine two problems of network equilibrium, with fixed and elastic demand, within some general formulation. The purpose of this article is to fill this gap.

We propose to consider a problem of network equilibrium with mixed demand which contains both constant and variable components. We show that it is a generalization of the problems of network equilibrium with fixed and elastic demand. In the article we formulate and prove the conditions of equilibrium and the conditions for existence of a solution, establish a connection with the problem of auction equilibrium. The results were announced at the conference [14].

1. NETWORK EQUILIBRIUM PROBLEMS WITH FIXED AND ELASTIC DEMAND

Let us recall the formulation of the network equilibrium problem with *fixed* demand. Let V be a set of nodes of the network, let A be a set of directed arcs of the network, and let W be a set of the origin—destination pairs (i, j) (or simply O/D-pairs), $i, j \in V$. For each $w \in W$, the set P_w of simple paths connecting w and the value of the fixed demand $d_w > 0$ are given. For convenience, we number all paths sequentially by the indices $p = 1, 2, \dots, \sum_{w \in W} |P_w|$.

The problem is to distribute the fixed demand d_w for each O/D-pair $w \in W$ among a given set of paths P_w, using an equilibrium criterion. We denote by x_p the variable value of the flow, passing along the path $p \in P_w$, $w \in W$. The admissible set

$$X = \left\{ x \mid \sum_{p \in P_w} x_p = d_w,\ x_p \geq 0,\ p \in P_w,\ w \in W \right\}$$

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